KMI-IEEC Joint International Workshop
"Inflation, Dark Energy and Modified Gravity in the PLANCK Era"

Black hole solutions and entropy in bigravity

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Based on Phys. Rev. D 87, 104032 (2013) arXiv: 1304.3181 TK and Shin'ichi Nojiri

In this talk

- We consider black hole solutions in bigravity for a minimal model.
- For the black hole solution, we evaluate the black hole entropy.
- In calculation for entropy, we use a recently proposed approach and we make it simple and useful to evaluate the entropy in bigravity.

Outline

- Introduction
- Black hole solutions in bigravity
- Black hole entropy in bigravity
- Summary

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Introduction

What is bigravity? ⟶ Generalization of massive gravity

Massive spin-2

- Linear massive gravity   Fierz and Pauli (1939)

  ↓ Vainshtein mechanism, Λ_3 theory etc.

- Non-linear massive gravity   de Rham, Gabadadze and Tolley (2011)

↓ General covariance (dynamical reference metric)

Massive spin-2
Massless spin-2

- Bigravity   Hassan and Rosen(2012)

If we consider the massless spin-2 field as graviton,
bigravity describes massive spin-2 field coupled to gravity.

Introduction

Five β_n s

- Mass : 1
- Cosmological constants : 2
- Free para. : 2

The action

$$S = M_g^2 \int d^4x \sqrt{-g} R(g) + M_f^2 \int d^4x \sqrt{-f} R(f)$$

Kinetic term of g — Kinetic term of f

$$+2m_0^2 M_{\text{eff}}^2 \int d^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f} \right)$$

Interaction term between g and f

Planck mass M_g, M_f

$$\frac{1}{M_{\text{eff}}^2} = \frac{1}{M_g^2} + \frac{1}{M_f^2}$$

$$e_0 = 1 \,, \quad e_1 = [X] \,, \quad e_2 = \frac{1}{2}\left([X]^2 - [X^2]\right) \,, \quad e_3 = \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right)$$

$$e_4 = \frac{1}{24}\left([X]^4 - 6[X]^2[X^2] + 3[X^2]^2 + 8[X][X^3] - 6[X^4]\right) \,, \quad [X] = X^{\mu}{}_{\mu}$$

Introduction

Bigravity describes massive spin-2 field coupled to gravity.

↓

Considering the black hole solution in bigravity, we obtain the black hole with massive spin-2 field.

↓ cf. Charged black hole in Einstein-Maxwell system.

- Massive spin-2 field can be "new hair" of black hole.
- How the massive spin-2 field near the horizon affects the black hole entropy?

Motivation

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Black hole solutions in bigravity

In this work, we consider the minimal model of bigravity

$$S = M_g^2 \int d^4x \sqrt{-g} R(g) + M_f^2 \int d^4x \sqrt{-f} R(f)$$
$$+2m_0^2 M_{\rm eff}^2 \int d^4x \sqrt{-g} \left(3 - {\rm tr}\sqrt{g^{-1}f} + \det\sqrt{g^{-1}f}\right)$$

with $\beta_0 = 3\,, \quad \beta_1 = -1\,, \quad \beta_2 = 0\,, \quad \beta_3 = 0\,, \quad \beta_4 = 1$

Equations of motion

$$0 = M_g^2 \left(\frac{1}{2} g_{\mu\nu} R(g) - R_{\mu\nu}(g)\right) + m_0^2 M_{\rm eff}^2 \left[\left(3 - \sqrt{g^{-1}f}\right) g_{\mu\nu} + \frac{1}{2} f_{\mu\rho} \left(\sqrt{g^{-1}f}\right)^{-1\rho}{}_{\nu} + \frac{1}{2} f_{\nu\rho} \left(\sqrt{g^{-1}f}\right)^{-1\rho}{}_{\mu}\right]$$

$$0 = M_f^2 \left(\frac{1}{2} f_{\mu\nu} R(f) - R_{\mu\nu}(f)\right) + m_0^2 M_{\rm eff}^2 \sqrt{\det(f^{-1}g)} \left[\det(\sqrt{g^{-1}f}) f_{\mu\nu} - \frac{1}{2} f_{\mu\rho} \left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} - \frac{1}{2} f_{\nu\rho} \left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\mu}\right]$$

$\longrightarrow$ It's hard to solve!

Black hole solutions in bigravity

We impose the condition. $\underline{f_{\mu\nu} = C^2 g_{\mu\nu}}$

On this condition, two e.o.m.s form

$$\begin{cases} 0 = R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) + \underline{\Lambda_g} g_{\mu\nu} \\ 0 = R_{\mu\nu}(f) - \frac{1}{2} f_{\mu\nu} R(f) + \underline{\Lambda_f} f_{\mu\nu} \end{cases}$$

Cosmological constants

For the consistency, both equations should be identical to each other. $\longrightarrow$ $C^2 = 1, \quad \Lambda_g = \Lambda_f = 0$

We obtain asymptotically flat solutions with $f_{\mu\nu} = g_{\mu\nu}$.

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Black hole entropy in bigravity

We use a recently proposed approach. Majhi and Padmanabhan (2011)

The basic idea :

Noether current from the surface term

↓ $$I_B = \frac{1}{16\pi G}\int_{\partial\mathcal{M}} d^{n-1}x\sqrt{\sigma}\mathcal{L}_B, \quad x^a \to x^a + \xi^a(x)$$

Virasoro algebra of the conserved charges

↓ $$Q_m, \quad i[Q_m, Q_n] = (m-n)Q_{m+n} + \frac{C}{12}m^2\delta_{m+n,0}$$

Using the Cardy formula $$S = 2\pi\sqrt{\frac{CQ_0}{6}}$$

Brown and Henneaux (1986)

Cardy (1986)

Carlip (1999)

Black hole entropy in bigravity

The surface term $\mathcal{L}_B = 2K(g) + 2K(f)$ K is Gibbons-Hawking term

N.B. We can ignore the interaction term, because it doesn't include derivatives and doesn't contribute to surface term.

$\longrightarrow$ It is simple and useful for calculation.

We use the Schwarzschild solutions.

$$ds^2 = -\frac{\rho}{\rho + 2M}dt^2 + \frac{\rho + 2M}{\rho}d\rho^2 + (\rho + 2M)^2(d\theta^2 + \sin^2\theta d\phi^2), \quad r = \rho + 2M$$

Diffeomorphism $\xi^t = T - (\rho + 2M)\partial_t T, \quad \xi^\rho = -\rho\partial_t T$

T is arbitrary function.

N.B. In this approach, we choose the diffeomorphism to keep the horizon structure invariant.

Black hole entropy in bigravity

We expand T. $$T = \sum_m A_m T_m, \quad A_m^* = A_{-m}$$

And we choose T_m so that the resulting ξ_m^a obeys the algebra isomorphic to Diff S^1.

$$i\{\xi_m, \xi_n\}^a = (m-n)\xi_{m+n}^a$$

{ , } is the Lie bracket.

Such a T_m is given by $$T_m = \frac{1}{\alpha}\exp[im(\alpha t + g(\rho) + p \cdot x)]$$

α is a constant, p is an integer and $g(\rho)$ is a function that is regular at the horizon

Substituting T_m into the Noether charges, we obtain the Fourier mode of the charges

Black hole entropy in bigravity

We obtain

$$Q_m = \underline{\frac{\kappa A}{4\pi\alpha G}}\delta_{m,0}\,, \quad [Q_m, Q_n] = -\frac{i\kappa A}{4\pi\alpha G}(m-n)\delta_{m+n,0} - im^3\frac{\alpha A}{8\pi\kappa G}\delta_{m+n,0}$$

Zero-mode eigenvalue

A is area and κ is the surface gravity

Virasoro algebra $$i[Q_m, Q_n] = (m-n)Q_{m+n} + m^3\underline{\frac{\alpha A}{8\pi\kappa G}}\delta_{m+n,0}$$

Central charge

From the Cardy formula $$S = 2\pi\sqrt{\frac{CQ_0}{6}} = \frac{A}{2G}$$

We obtain a double portion of Bekenstein-Hawking entropy.

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Summary

- We have shown that the bigravity for the minimal model has asymptotically flat solutions with $f_{\mu\nu} = g_{\mu\nu}$.
- We have evaluated the black hole entropy for the Schwarzschild solution.
- We find that the obtained entropy is twice as much as the Bekenstein-Hawking entropy in the Einstein gravity.
- It is interesting that our approach may be generalized to the case of the other models.